**N15** USE INEQUALITY NOTATION TO SPECIFY SIMPLE ERROR INTERVALS DUE TO TRUNCATION OR ROUNDING(foundation and higher tier)

You should be able to find least and greatest value of a number that has been rounded to the nearest whole number.

e.g. 10 km measured to the nearest km lies between 9.5 km and 10.5 km.

e.g. The volume of a cylinder is 24 cm3 to the nearest cm3.

The least possible volume is 23.5 cm3 and the greatest possible volume is 24.5 cm3.

**Error Intervals.**

Once you have found the least possible value and the greatest possible value you can write down the **error interval**.

For a value *x*, the **error interval** is: least possible value ≤ *x* < greatest possible value

**EXAMPLE 1**

A fence is 30 m long to the nearest 10 m.

Write down the error interval for the length, *l*.

Here we have to think of all the numbers that can be rounded up to 30.

A number line can be useful to observe these numbers.

30

35

25

20

40

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

The least possible value of *l* is 25 m To the nearest 10 so between 20 and 30

30

35

25

20

40

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

The greatest possible value of *l* is 35 m To the nearest 10 so between 30 and 40

The error interval for the length is written as

25 m ≤ *l* ˂ 35 m

**NOTE:** We do not write the error interval as 25 m ≤ *l* ˂ 34.99999999999.... m as we want to know the greatest possible value that *l* is less than (and not equal to).

For our error interval we have *l* < 35 and not *l* ≤ 35 as *l* cannot equal 35

**EXAMPLE 2**

The width, *w*, of a stool is 243.6 mm correct to one decimal place.

Write down the error interval for the width of the stool.

243.55

243.65

\_\_\_\_\_\_\_\_

243.5

243.7

243.6

\_\_\_\_\_\_\_\_

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

The least possible value of *w* is 243.55 mm To 1 d.p. so between 243.5 and 243.6

The greatest possible value of *w* is 243.65 mm To 1 d.p. so between 243.6 and 243.7

The error interval for the width is written as

243.55 mm ≤ *w* ˂ 243.65 mm

**EXERCISE 1:**

1. The weight of a bag of potatoes is 35 kg, correct to the nearest kg.

(a) Write down the smallest possible weight of the bag of potatoes.

(b) Write down the largest possible weight of the bag of potatoes.

2. The length of a line is 73 centimetres, correct to the nearest centimetre.

(a) Write down the **least** possible length of the line.

(b) Write down the **greatest** possible length of the line.

3. The weight, *w*, of a suitcase is 14 kg, correct to the nearest kilogram.

Write down an inequality to show the error interval for *w*.

4. Katy drove for 238 miles, correct to the nearest mile.

(a) Write down an error interval for the number of miles (*m*) driven.

Katy used 27.3 litres of petrol, to the nearest tenth of a litre.

(b) Write down an error interval for the number of litres (*l*) used.

5. The length, *l*, of a piece of wood is 123 mm, correct to the nearest mm.

Write down an inequality to show the error interval for *l*.

6. Sandeep takes 35 seconds, to the nearest second, to run a race.

Write down an error interval for the time. *t* seconds, taken to run the race.

7. The length *L* of a piece of string is 16 cm, correct to the nearest cm.

Write down an error interval for *L*.

8. The number *x*, rounded to one decimal place, is 5.7

(a) Write down the error interval for *x*.

The number *y*, rounded to two decimal places, is 7.36.

(b) Write down the error interval for *y*.

The number *z*, rounded to two significant figures, is 5800.

(c) Write down the error interval for *z*.

9. The perimeter of a square is 48 cm to the nearest centimetre.

Work out the error interval of the length, *l*, of a side of this square.

10. A pencil is 12 cm long to the nearest centimetre.

Tom says that this pencil will always fit in a pencil case that is 12.3 cm.

Is Tom correct? Explain your answer.

11. A number *x* has three non-zero digits after the decimal point.

When *x* is rounded to two decimal places it is 2.75

When *x* is rounded to one decimal place it is 2.8

Work out the range of values for *x*.

12. James was timed at 60.74 seconds, to the nearest hundredth of a second, for driving

one circuit of a racing track.

(a) State the lower bound of the time.

(b) Hence, write down the error interval for the time, *t* seconds.

13. Sam measured the distance between two points as 18.7 cm to the nearest mm.

Write down the error interval for the distance, *d* cm.

14. Fifty identical books are stacked in one pile. The height of each book is 7 cm, to the nearest cm.

Work out the maximum height of the pile.

15. A small van can take a load of up to 1500 kg.

This van has to take the following to a market:

50 large bags of potatoes, each bag weighing 25kg to the nearest kg,

240 boxes of oranges, each box weighing 3000 g to the nearest 10 g.

Could this load be over the limit?

You must show all your working.

**Truncation**

Truncation is a method of approximating a decimal number by dropping all decimal places past a certain point without rounding.

**EXAMPLE 3**

Truncate 3.14159265 to 4 decimal places.

3.1415 9265 First draw a vertical line after 4 d.p.

Answer is 3.1415 Remove all the digits after the vertical line

**NOTE**: 3.14159265 rounded to 4 decimal places is 3.1416

**EXAMPLE 4**

Truncate 782.9 to the whole number.

782. 9 First draw a vertical line by the decimal point

Answer is 782 You do not need to put in the decimal point

**NOTE**: 782.9 rounded to the nearest whole number is 783

**EXAMPLE 5**

A length, *l*, is 97.62 truncated to two decimal places.

Write down the error interval for the length *l*.

The least possible value is 97.62 It has already been truncated so it cannot be less than 97.62

The greatest possible value is 97.63 It has been truncated so the 2nd decimal place must be less than 3

The error interval is 97.62 ≤ *l* ˂ 97.63 Remember to use ...... ≤ *l* < ......

**Problems with upper and lower boundary**

You also need to consider more complex problems using greatest and least values.

**EXAMPLE 6**

Ravina measured the length and the width of her rectangular garden.

She measured the length to be 15 m correct to the nearest metre and the width to be 9.28 m correct to the nearest centimetre.

(a) Write down the error interval for the length, *l* metres.

(b) Write down the error interval for the width, *w* metres.

(c) Hence, or otherwise work out the error interval for the perimeter (*P* metres) of   
 the rectangle.

(a) The least possible value is 14.5 m To nearest metre so least is between 14 and 15

The greatest possible value is 15.5 m and greatest is between 15 and 16

The error interval is 14.5 m ≤ *l* < 15.5 m

(b) The least possible value is 9.275 m To nearest cm (i.e. to 2 decimal places)

The greatest possible value is 9.285 m so least is between 9.27 and 9.28

The error interval is 9.275 m ≤ *w* < 9.285 m and greatest is between 9.28 and 9.29

(c) Least possible perimeter

= 14.5 + 14.5 + 9.275 + 9.275 Add the 4 sides using least length and least width

= 47.55 m

Greatest possible perimeter Add the 4 sides using greatest length

= 15.5 + 15.5 + 9.285 + 9.285 and greatest width

= 49.57 m

error interval for the perimeter is 47.55 m ≤ *P* < 49.57 m

**EXERCISE 2:**

1. Truncate 4.87635

(a) to 2 decimal places

(b) to 4 decimal places

(c) to the whole number

2. The length, *l* cm, of cable is 74.3 m truncated to 1 decimal place.

Write down the error interval for the length *l*.

3. The length, *L* cm, of a line is measured as 13 cm truncated to the whole number of centimetres.

Write down the error interval for the length.

4. Jim truncates the number *x* to one decimal place.

The result is 7.2

Write down the error interval for *x*.

**EXERCISE 3:**

1. Steve measures the length (*l*) and the width (*w*) of a rectangle.

He measures the length to be 640 mm correct to the nearest 10 mm.

(a) Write down the error interval for the length.

Steve also measures the width to be 400 mm correct to the nearest 100 mm.

(b) Write down the error interval for the width.

(c) Hence, or otherwise work out the error interval for the perimeter, *P*, of the rectangle.

2. A field is in the shape of a rectangle.

The length of the field is 340 m, to the nearest metre.

The width of the field is 117 m, to the nearest metre.

Work out the error interval for the perimeter, *p*, of the field.

3.Mia’s weight is 57 kg, correct to the nearest kilogram.

(a) Write down

(i) her least possible weight (*w* kg),

(ii) the error interval of her weight.

Alice’s weight is 62 kg, correct to the nearest kilogram.

(b) Work out the greatest possible difference between Alice’s weight and Mia’s weight.

**ANSWERS**

**Exercise 1**

1. (a) 34.5 (b) 35.5

2. (a) 72.5 (b) 73.5

3. 13.5 ≤ *w* ˂ 14.5

4. (a) 237.5 ≤ *m* ˂ 238.5 (b) 27.25 ≤ *l* ˂ 27.35

5. 122.5 ≤ *l* ˂ 123.5

6. 34.5 ≤ *t* ˂ 35.5

7. 15.5 ≤ *L* ˂ 16.5

8. (a) 5.65 ≤ *x* ˂ 5.75 (b) 7.355 ≤ *y* ˂ 7.365 (c) 5750 ≤ *z* ˂ 5850

9. 11.875 ≤ *l* ˂ 12.125

10. ‘No’ as the pencil could be longer than 12.3 cm e.g. 12.4 cm

11. 2.745 ≤ *x* ˂ 2.85

12. (a) 60.735 (b) 60.735 ≤ *t* ˂ 60.745

13. 18.65 ≤ *d* ˂ 18.75

14. 375 cm

15. Yes as Upper bound = (50 × 25.5) + (240 × 3.005) = 1996.2 kg

**Exercise 2**

1. (a) 4.87 (b) 4.8763 (c) 4

2. 74.3 ≤ *l* ˂ 74.4

3. 13 ≤ *L* ˂ 14

4. 7.2 ≤ *x* ˂ 7.3

**Exercise 3**

1. (a) 635 ≤ *l* ˂ 645 (b) 350 ≤ *w* ˂ 450 (c) 1970 ≤ *P* ˂ 2190

2. 912 ≤ *p* ˂ 916

3. (a) (i) 56.5 (ii) 56.5 ≤ *w* ˂ 57.5 (b) 6